



The Study of Topological Properties and its Application

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Abstract : *Topology being a new field of research deals with all abstract properties of shapes (not in a literal sense).*

We all are curious to know something new in our life, and topology follows that curiosity. What do we mean by the concept of 'shape'? What does it mean to say that an object has a 'hole' in it? And is the hole in the centre of a basketball net the same as the hole in the centre of a doughnut? What a rigorous and mathematical means of establishing that two objects are equivalent- from the point of view of shape and form? These are the types of questions that we consider in the subject of topology. Dawn topology can be considered by Euler's published paper in 1736 on the solution of the Königsberg bridge problem. After that topology found its genesis in the mathematical works of Jules Henri Poincare (1854-1912).

The axioms are merely a springboard for "rubber sheet geometry". By abstracting the notion of openness, we can now make sense of continuity in a way that's more general than what we might learn in calculus classes. Continuity is an essential ingredient to the bending, twisting, and stretching— but not tearing— of shapes that we hear about. The 'no thearing' part is precisely what continuity means. From the abstract properties of topology we studied the wonderful structure of Moebius strip and Kleinbottle.

Keywords: *Topological spaces, compactness, connectedness, topological equivalent, Moebius strip, Klein bottle*

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Introduction :

Topology is a newly born child of twentieth century's mathematical thinking. It has been allowed to consider the shape and structure of an object without being wedded to its size or to the distance between its component parts. Topology is by its very nature abstract. Traditionally it follows the Euclidean paradigm of formulating axioms and definitions and then proving theorems. Topology found its genesis in the mathematical works of Jules Henri Poincare. It has been a burgeoning part of mathematics ever since. The first actual use of the word "topology" was in the treatise *Vorstudieren zur Topologie* by Johann Listing (1808-1882). In the past 50 years topology has played an

increasingly prominent role in theoretical physics. Engineers and even theoretical computer scientists take advantage of topological thought in developing their ideas. A very recent development is that the modern theory of data mining is founded in topology (G. Krantz Steven, 2015).

Topology can be considered as an abstract side of geometry. In geometry we talk about the differences between two shapes in terms of distance, area, volume, angles, etc., but in topology we are free to transform one shape to another until and unless it does not get torn, cut, or joined. There are some short of rules and properties which can make two geometrically different shapes as topologically equivalent.

The more we deal with topological spaces, the more we discover things like a surface with only one side or the fact that it's possible to turn pants inside out without taking the feet off the ground. On the applied side, topology is used in neuroscience, data analysis and signal processing, fluid dynamics, string theory, and topological quantum field theory. There has been considerable fusion between topology and algebra, giving rise to algebraic topology.

Objectives :

1. To understand and make an intuitive study on topological properties.
2. To study some interesting shapes like Moebius strip and Kleinbottle.
3. To show the application of topology in Digital Imaging, Networking.

Methodology :

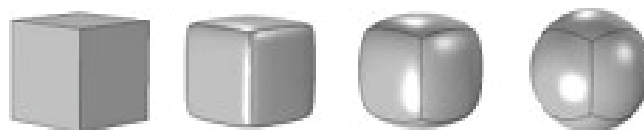
In this project we have dealt with the abstract properties in topology. How homeomorphism is important to call two geometrically different shapes topologically same. We studied several books related to topology, some journals and then we tried to generalize the concept and basic theory with some intuitive way. Further, we experimented by ourselves to understand Moebius strip more precisely. After consulting several books, journals and web surfing we initiated to make topology our topic of research.

Our secondary sources were websites, books and journals.

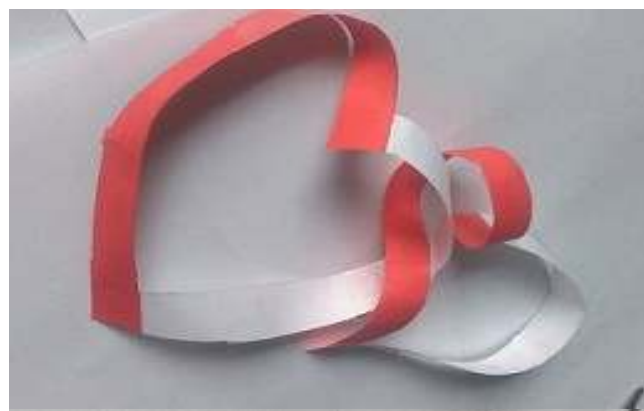
Topological Equivalence or Homeomorphism

According to Euclidean geometry, equivalence between two geometrical figures is measured by their congruence and similarity. While the congruent figures are essentially similar, the converse is not true. Congruence measures the equivalence of figures in all respect except the position in space while similarity measures the equivalence of shape and not of size. According to projective geometry, the shapes and sizes of equivalent figures may or may not be the same but they should possess some certain common properties e.g., a straight line in one figure should correspond to the straight line in an equivalent figure.

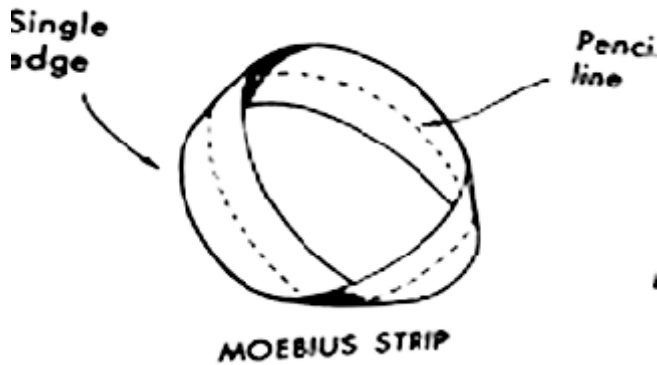
In topology the equivalence of figures is measured by the property that each one can be transformed into another by a continuous deformation, e.g., a piece of plasticine moulded into different shapes as a sphere, an ellipsoid, a prism, a cube etc., without cutting or tearing the piece, and gives equivalent surfaces (Gupta B.D., 2011).



Structure of Moebius strip :



If we take a flat strip of paper instead of a cylinder, this unsurprisingly fact can be frustrated. If a half twist given to the strip before joining the ends, we will have two opposite sides joined. Also the 2 opposite edges is called a Moebius strip. If we follow the edge, we get to know that it is now in one continuous line which joins itself, forming a loop. And if we make an ant crawl on the strip then the ant will come back to its own beginning.



The Moebius strip has 1 side and 1 edge. This is a new kind of surface, and it has a new kind of connectivity.

Structure of Kleinbottle : The place where Kleinbottle intersect is arbitrary, and there are no points on the curve of intersection that are on two different parts of the surface at once, i.e., on both the narrow neck and the main body. Kleinbottle is non-orientable. We could tell it was, anyway, from its construction – or connectivity. But unlike Moebius strip, it has no edges: both pairs have been joined, and consequently effaced. This is the reason for insisting that the Kleinbottle – and, when we come to it, the projective plane- must have all of both pairs of edges joined.

Conclusion :

The research was focused on curiosity to explore more in the field of pure mathematics. Topology, being a very new and vast branch of mathematics, deals with curiosity and imagination. From some abstract properties which have been taught in point set topology, we can think critically and present the ideas of some simple but complex structure, intuitively. To be precise, we can conclude that, although we are able to understand the complexities of those structures, that is practically impossible in 3-dimensional space. Some say it is possible to do in a 4- dimensional space. The result of studying these shapes is that researchers are able to explain the complexities in DNA (studying knots, manifolds). Using the theory of topological spaces, Digital imaging can be explained, Graph theory can be understood, Networking can be structured and data can be analyzed. This is just a small study in topology.

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